Vacuum Polarization Effects in the Worldline Variational Approach to Quantum Field Theory

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with

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1. Introduction: worldline variational approach

2. Beyond the quenched approximation

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4. Summary

1. Introduction

Quantum Mechanics:

operators, states in Hilbert space Heisenberg, Schrödinger etc.	$\longleftrightarrow$	path integrals, trajectories Dirac, Feynman
" WAVES "	$\longleftrightarrow$	" PARTICLES "

Field Theory:

field operators, states in Fock space Jordan, Heisenberg, Pauli etc.	$\longleftrightarrow$	worldlines $x_\mu(t)$ Feynman ($\sim$ 1950)
" FIELDS "	$\longleftrightarrow$	" PARTICLES "
" **second quantization** " $\Downarrow$ wins ! (see textbooks)	$\longleftrightarrow$	" **first quantization** " $\uparrow$ renaissance ... from string theory (!) Bern & Kosower (1991)

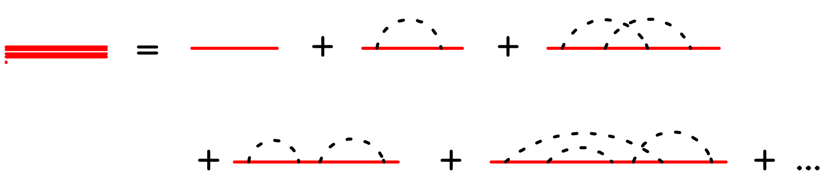
Strassler (1992) showed how to derive the Bern-Kosower rules from the particle (worldline) representation of Quantum Field Theory

Advantages:

a) efficient way to calculate diagrams with many legs

b) new approximation methods for large couplings – as in the **polaron** problem : Feynman (1955)

best analytical method which works for **all** coupling constants !



sums (approximately) self-energy diagrams:

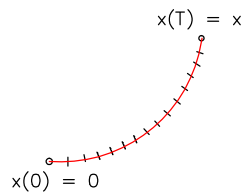
Steps in the worldline variational approach:

1. Integrate out heavy particles

2. **Schwinger representation**:

$$\frac{1}{\hat{\mathcal{O}} + i0} = -i \int_0^\infty dT \, e^{i T \hat{\mathcal{O}}} \, , \qquad\qquad T \; : \; \text{proper time}$$

3. Write matrix element as (**quantum mechanical**) path integral over particle trajectory $x_\mu(t)$:



4. Integrate out light particles : only possible in

quenched approximation $\longrightarrow$ retarded (two-time) action

5. **Feynman-Jensen variational principle**: $\left\langle e^{-(S-S_t)} \right\rangle \geq e^{-\left\langle (S-S_t) \right\rangle}$

trial action S_t = retarded quadratic action

6. Solve variational equations for parameters/functions in trial action

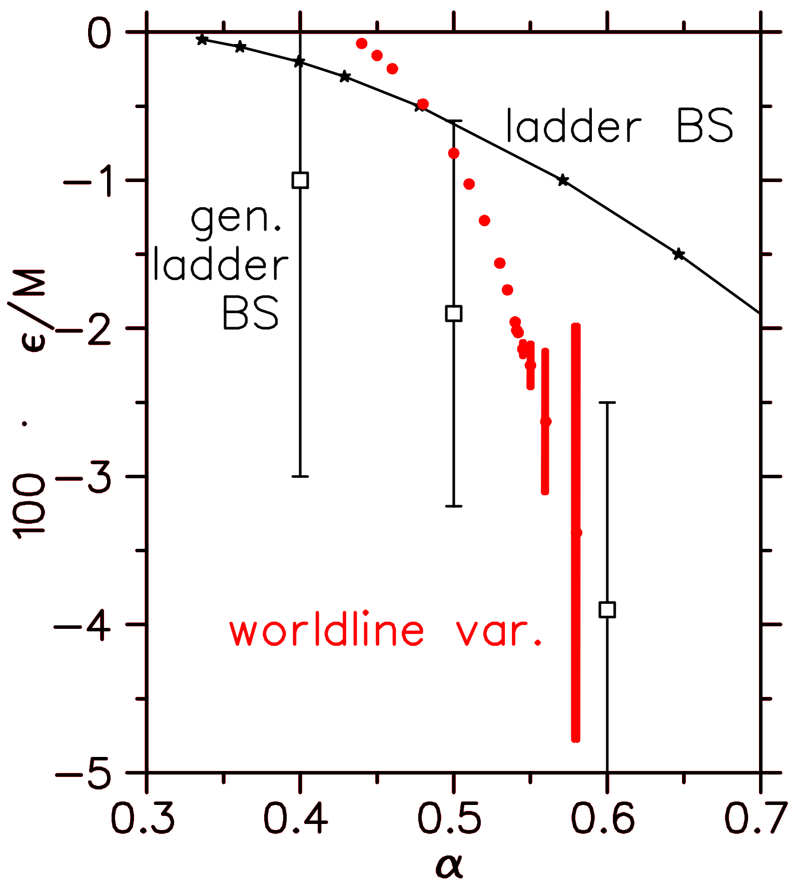
$\Longrightarrow$ result includes (approximately) all self-energy and vertex corrections !

Applications

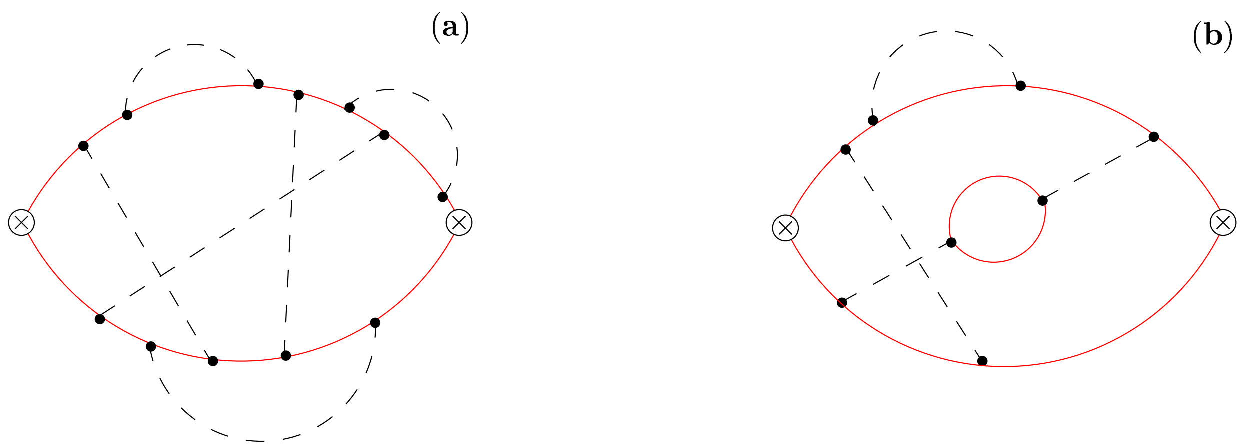
- Full propagator in quenched scalar model
  RR & Schreiber, Phys. Rev. **D53** (1996)
- Processes with one/two external mesons
  Alexandrou, RR & Schreiber, Nucl. Phys. **A 601** , **A 628** (1998)
  deep-inelastic inclusive scattering
  Fettes & RR, Few-Body Syst. **24** (1998)
- Improved (anisotropic) trial actions
  Schreiber & RR, Eur. Phys. J. **C 25** (2002)
- quenched QED: non-perturbative expression for anomalous mass dimension of electron
  Alexandrou, RR & Schreiber, Phys. Rev. **D 62** (2000)
  Abraham-Lorentz-like equation for electron
  RR & Schreiber, Eur. Phys. J. **C 37** (2004)
- Relativistic bound-state problem in quenched scalar model
  Barro-Bergflödt, RR & Stingl, Mod. Phys. Lett. **A 20** (2005)

Numerical solutions of var. eqs.: have found pole below $q^2 < 4M^2$

binding energy ϵ vs. coupling constant $\alpha = \frac{g^2}{4\pi M^2}$:



typical diagrams:



quenched

unquenched

2. Beyond the quenched approximation

Up to now **vacuum polarization** (VP) terms had to be neglected because

1. Number of worldlines for heavy particles is conserved (physics)

2. Meson/photon field cannot be integrated out with VP effects included (mathematics)

example: scalar Wick-Cutkosky model with $\mathcal{L}_{\text{int}} = g \sum_i^N |\Phi_i|^2 \chi$

Φ_i : "nucleon" field with N species, χ : "meson" field

functional integral over meson field χ contains determinant

$$D[\chi] = \left[\text{Det} \left(-\partial^2 - M_0^2 - 2g\chi \right) \right]^{-N/2}$$

How to include VP effects?

First possibility: expand

$$\frac{D[\chi]}{D[0]} = \exp\left\{ -\frac{N}{2} \mathrm{Tr} \ln \left[1 + \frac{2g}{\partial^2 + M_0^2} \chi \right] \right\}$$

$$\approx \exp\left\{ -\frac{N}{2} \mathrm{Tr} \left[2g \frac{1}{\partial^2 + M^2} \chi - \frac{(2g)^2}{2} \frac{1}{\partial^2 + M^2} \chi \frac{1}{\partial^2 + M^2} \chi + \ldots \right] \right\}$$

to second order in the meson field ($M_0 \approx M$)

linear terms in χ: tadpoles (irrelevant)

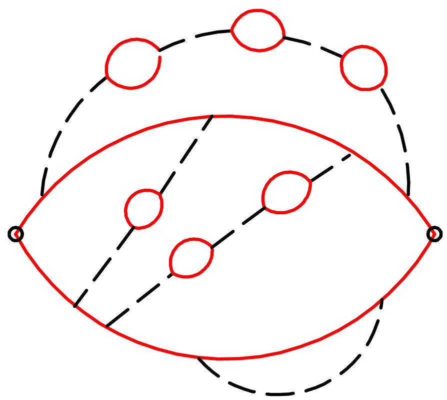
quadratic terms in χ:

- meson mass renormalization: $m_0 \to m$
- modification of meson propagator by (renormalized) one-loop VP contribution

$\Longrightarrow$ n-particle worldline action

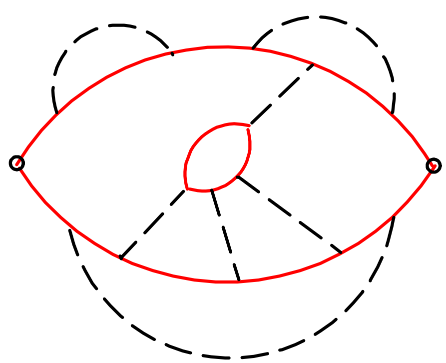
$$S = \underbrace{\sum_{i=1}^{n} \int_0^{T_i} dt \left(-\frac{1}{2} \dot{x}_i^2 \right)}_{\text{free action}} - \underbrace{\frac{g^2}{2} \sum_{i,j=1}^{n} \int_0^{T_i} dt \int_0^{T_j} dt' \int \frac{d^4k}{(2\pi)^4} \frac{\exp\left[-ik \cdot (x_i(t) - x_j(t')) \right]}{k^2 - m^2 - \pi_r(k^2)}}_{\text{retarded interaction}}$$

Contains VP insertions in all meson lines:



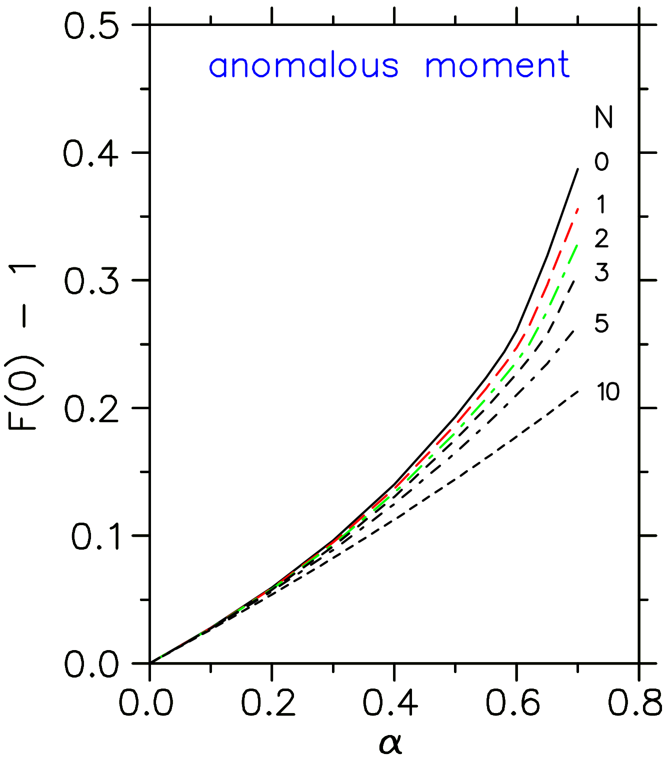
main effect :
$$\alpha \longrightarrow \alpha^{\star} \;=\; \frac{\alpha}{1+\frac{\alpha}{6\pi}\underbrace{N}_{\text{number of flavors}}}$$

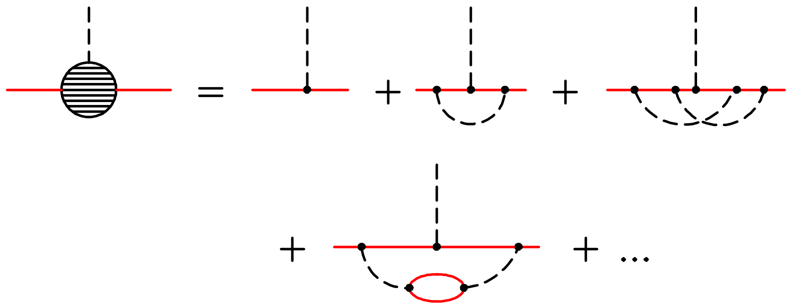
but **no** interaction of pair-produced particles:



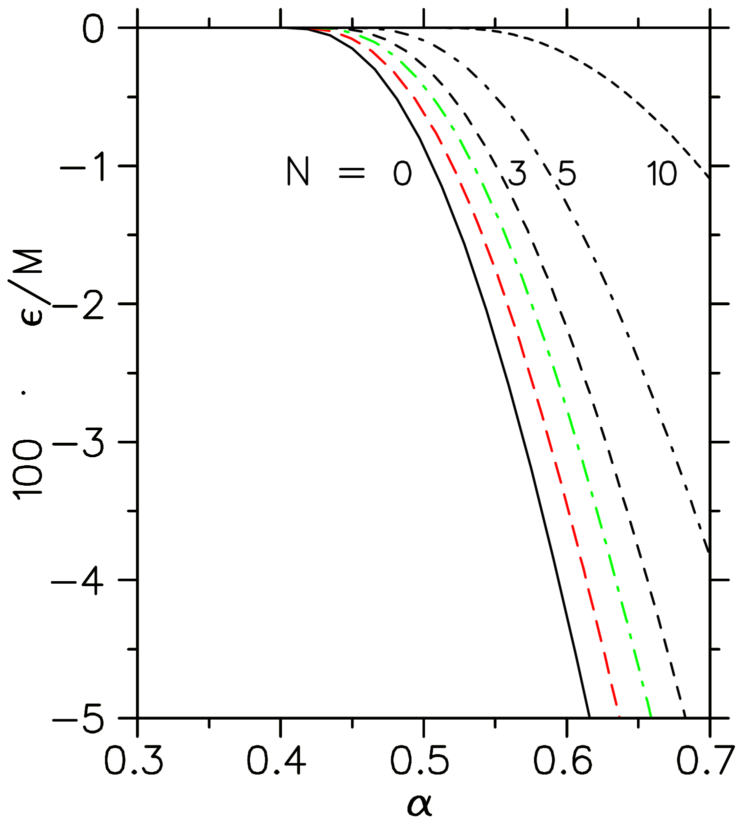
3. Results

Anomalous moment vs. coupl. constant for different N :





Binding energy vs. coupl. constant for different N :



Second possibility:

"hybrid" approach: apply variational principle for **both** meson field χ and worldline $x(t)$

as in the **linear polaron model** Bogoliubov (1978)

$$S_t[x,\chi] \;=\; \text{quadratic in } x(t) \;+\; \underbrace{\text{quadratic in } \chi}_{}^{\text{modified meson propagator}\ \downarrow} + \int_0^T dt \int d^4k \;\underset{\uparrow\ \text{variational coupling function}}{\ell(k^2)}\; ik\cdot x(t)\,\chi(k)$$

solvable trial action, averages can be evaluated,

(numerical) results in progress

4. Summary

Worldline variational methods for field theories

- are successful due to reduction in number of variables
  e.g. $\Phi(x), \chi(x) \quad \longrightarrow \quad x(t)$
- have been applied to scalar and fermionic theories
  give gauge-invariant results
- offer a new approach to the relativistic bound-state problem which includes self-energy effects and vertex corrections consistently
- can include vacuum-polarization effects

Possible extensions:

- ◦ improved (anisotropic) trial action
- ◦ systematic second-order corrections to variational result
- ◦ inclusion of other internal degrees of freedom (color) in worldline formalism.
  QCD ?

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